Important Concepts in Electromagnetics

Coulomb's Law

Gauss's Law

Maxwell's equations

Static Boundary Conditions

Time Harmonic Maxwell's equations

Traveling waves

Hemholtz Equation

Time Harmonic Boundary Conditions

Transmission Lines

Gauss's Law + Divergence Theorem

$$\boxed{\oint \bar{D} \cdot d\bar{S} = Q_{enc} \quad \text{Gauss's Law}}$$

where $\boxed{Q_{enc} = \int_V \rho_v dv}$, or $\oint_S \rho_s ds$, or $\int_l \rho_l dl$

Net electric flux $\Psi = Q$

<u>Note:</u> $\oint \bar{D} \cdot d\bar{S} = |\bar{D}| \oint d\bar{S} = Q_{enc} = \int_V \rho_v dv$

$\Rightarrow |\bar{D}| = \ldots$ done

- this is how we solve for E using a Gaussian surface

$\nabla \cdot \bar{D} = \rho$ pt form of Gauss's

<u>Divergence Theorem:</u>

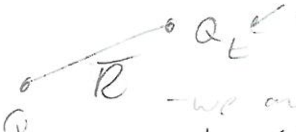
$\oint \bar{D} \cdot d\bar{S} = \int (\nabla \cdot \bar{D}) dv$, from Gauss's $\oint \bar{D} \cdot d\bar{S} = \int \rho dv = Q$

Coulomb's Law

- coulomb's law is the force between 2 charges

$$\boxed{F_1 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 |R_{21}|^2}\hat{a}_{21}}$$

- Electric feild from coulomb's due to a pt. charge



- we are finding the E feild from Q at the pt on the test charge

$$\bar{E} = \frac{\bar{F}_t}{Q_t} = \frac{1}{Q_t}\left[\frac{Q Q_t}{4\pi\varepsilon_0 |\bar{R}|^2}\bar{r}\right]$$

$$\boxed{\bar{E} = \frac{Q}{4\pi\varepsilon_0 |\bar{R}|^2}\bar{r} \text{ due to a point charge}}$$

- charge distributions

$$\rho = \frac{dQ}{dv}\left(\frac{C}{m^3}\right) \Rightarrow dQ = \rho v \Rightarrow Q = \int \rho\, dv$$

$$dE = \frac{dQ}{4\pi\varepsilon_0 R^2}\hat{r} \Rightarrow \boxed{\bar{E} = \int \frac{\rho\, dv}{4\pi\varepsilon_0 R^2}\hat{r}}$$

also works for line or surface

Displacement Current and Induced EMF

- from Ampere's law we know that

$$\Phi = \int \bar{B} \cdot d\bar{S}$$ is the mag flux

- due to displacement current and Induced EMF

$$V = -\frac{d\Phi}{dt}$$

- scalar potential

$$V = \int \bar{E} \cdot d\bar{\ell}$$

Laplace's Equation

- gives you voltage as a func of distance
- 1-D version

$$\frac{d^2V}{dy^2} = 0$$

- integrate both sides to get

$$V = Ay + B$$

- you can find

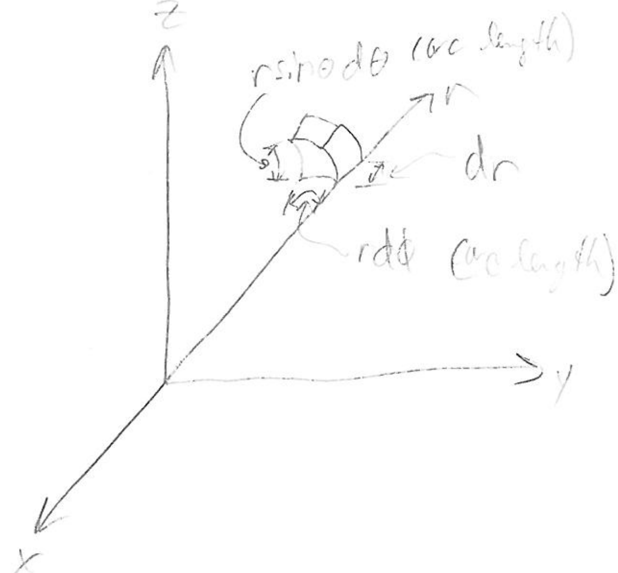
$E = -\nabla V = -\nabla[Ay + B]$ to get solve for A and B

- Or, if given voltage potentials at various locations, just subst. them into

$V = Ay + B$

A Note on Differential Components

- spherical coords



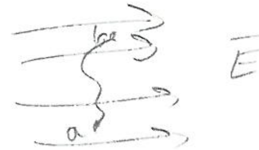
- you can make any $d\bar{S}$, dV comp's from this

ie $d\bar{S}_r = \hat{r}\, r^2 \sin\theta\, d\phi\, d\theta$

- where $\hat{r}$ is the unit normal vector to the surface

Electric Feild Potential

$$\boxed{V_{ab} = -\int_b^a E \cdot dl} \quad (V)$$



Potential of a charge distribution:

$$\boxed{V = \int_V \frac{\rho\, dv}{4\pi\varepsilon_0 R}}, \quad \text{because } dV = \frac{dQ}{4\pi\varepsilon_0 R}$$

E-feild and Potential

$$\boxed{E = -\nabla \overline{V}}$$

Potential due to a pt. charge

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0 r}}$$

Current Density

the continuity equation: $\nabla \cdot \bar{J} = -\frac{\partial \rho}{\partial t}$

current eq: $I = \int_S \bar{J} \cdot d\bar{S}$

Capacitance and Polarization

Polarization: a function of susceptability χ_e

$$\boxed{\bar{P} = \chi_e \varepsilon_0 E,}$$ where $\varepsilon_r = 1 + \chi_e$

$\Rightarrow \chi_e = \varepsilon_r - 1$

$$\bar{D} = \varepsilon_0 \varepsilon_r E$$

Capacitance: $\boxed{C = \frac{Q}{V}}$

Parallel plate cap: $\boxed{C = \frac{\varepsilon_r \varepsilon_0 A}{d}}$

Static Boundary Conditions

Boundary conditions:

1) tangential component of $\bar{E}$ is continuous accross a dielectric layer

$$\boxed{E_{t1} = E_{t2} \quad , \quad \frac{D_{t1}}{\varepsilon_{r1}} = \frac{D_{t2}}{\varepsilon_{r2}}}$$

2) normal component of $\bar{D}$ is cont. accross a dielectric layer

$$\boxed{D_{n1} = D_{n2} \quad , \quad \varepsilon_{r1} E_{n1} = \varepsilon_{r2} E_{n2}}$$

- if a charge density exists on boundary, then

$$\boxed{D_{n1} - D_{n2} = -\rho_s \quad , \quad \varepsilon_{r1} E_{n1} - \varepsilon_{r2} E_{n2} = \frac{-\rho_s}{\varepsilon_0}}$$

Ampere's Law and Magnetic Field

Biot-Savart's: $\bar{H} = \oint \frac{I d\bar{l} \times \hat{R}}{4\pi R^2}$

Ampere's Law: $I_{enc} = \oint \bar{H} \cdot d\bar{l}$

Relationship Between $\bar{J}$ and $\bar{H}$: $\bar{J} = \nabla \times \bar{H}$

Magnetic Flux Density: $\bar{B} = \mu \bar{H}$

Magnetic Flux: $\Phi = \int_S \bar{B} \cdot d\bar{S}$

Magnetization: $\bar{B} = \mu_0 \bar{H} + \bar{M}$

$\bar{M} = \chi_m \mu_0 \bar{H}$

Vector Magnetic Potential: must satisfy: $\begin{cases} \nabla \times \bar{A} = \bar{B} \\ \nabla \cdot \bar{A} = 0 \end{cases}$ (find $\bar{B}$ first, then solve the diff eq for $\nabla \times \bar{A}$)

current filament: $\bar{A} = \oint \frac{\mu I d\bar{l}}{4\pi R}$

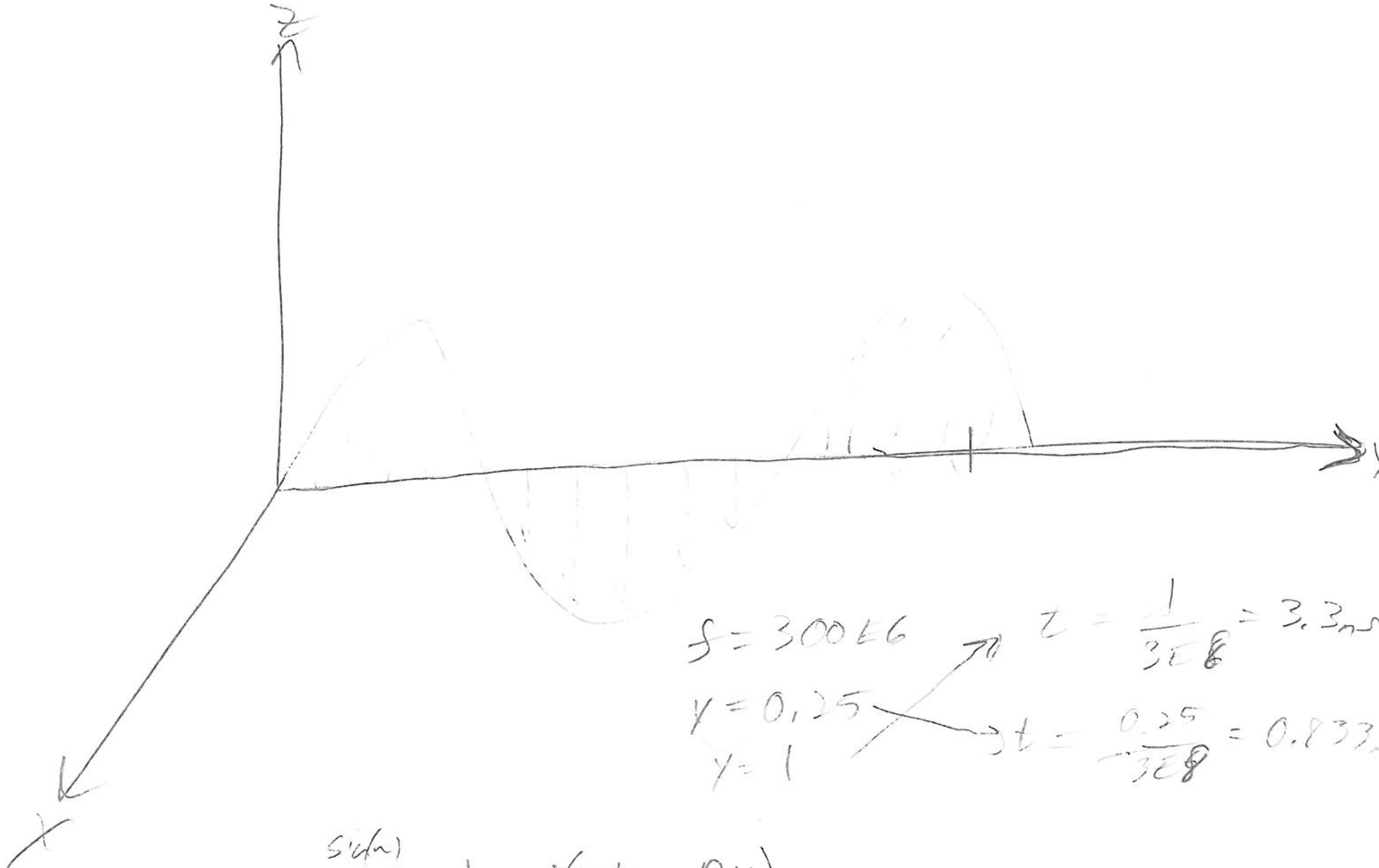
sheet current: $\bar{A} = \int_S \frac{\mu \bar{K} d\bar{S}}{4\pi R}$

volume current: $\bar{A} = \int_V \frac{\mu \bar{J} d\bar{v}}{4\pi R}$

Stoke's Theorem: $\oint \bar{A} \cdot d\bar{l} = \int (\nabla \times \bar{A}) \cdot d\bar{S}$

- a way of finding the surface integral of a curl by using a single line integral

Electromagnetic waves



$f = 300E6$

$y = 0.25$

$y = 1$

$t = \frac{1}{3E8} = 3.3ns$

$t = \frac{0.25}{3E8} = 0.833ns$

$$\bar{E} = \hat{z}|E|\, e^{j(\omega t - \beta y)}$$

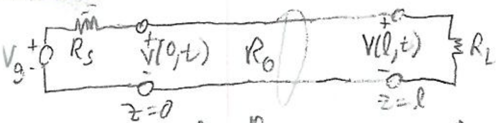
$$\cos(\omega t - \beta y) + j\sin(\omega t - \beta y)$$

$1.88E9 \qquad 6.283$

$$1\, e^{j\left[2\pi 300E6(t) - j\frac{2\pi 300E6}{3E8}(y)\right]}$$

$-13.1E-3 + j4.81$

BOUNCE DIAGRAMS



$$\Gamma_L = \frac{R_L - R_0}{R_L + R_0} \qquad \Gamma_S \; \frac{R_S - R_0}{R_S + R_0}$$

$T = \frac{L}{V}$ 1-way transit time

$0 < t < T \qquad V(z,t) = \frac{R_0}{R_0 + R_S} V_g(t - z/v)$

for initially charged T-lines, use

$$V = \frac{-V_0 R_0}{R_L + R_0}$$

where

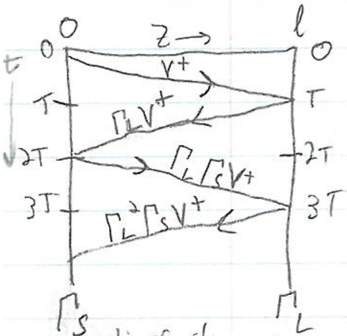
Reversed Bounce diagram

Rev Γ_L ad Γ_S

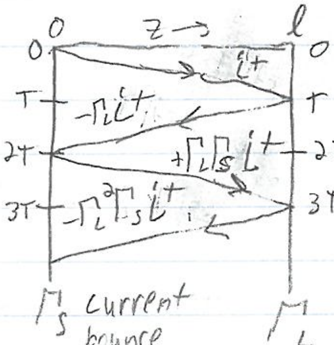
$T < t < 2T \qquad V(z,t) = \frac{R_0}{R_0 + R_S}\left[V_g(t - z/v) + \Gamma_L V_g(t - 2T + z/v)\right]$

$2T < t < 3T \quad V(z,t) = \frac{R_0}{R_0 + R_S}\left[V_g(t - z/v) + \Gamma_L V_g(t - 2T + z/v) + \Gamma_L \Gamma_S V_g(t - 4T + z/v)\right]$

etc...



voltage bounce Diagram, follows leading edge



current bounce diagram

$$i^+ = \frac{V^+}{R_0}$$

$$V^+ = \frac{V_0 R_0}{R_0 + R_S}$$

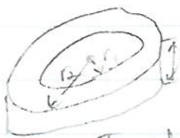
Inductance and Magnetic Circuits:

Inductance: flux linkage $= \lambda = N\Phi$ where $N =$ # turns, $\Phi =$ mag flux

inductance $= L = \frac{\lambda}{I}$ where I is the coil current

Standard Conductor Configurations:

Toroid, square cross section:



$$L = \frac{\mu_0 N^2 a}{2\pi} \ln\left(\frac{r_2}{r_1}\right) \quad (H)$$

Toroid, general cross section S:

$$L \approx \frac{\mu_0 N^2 S}{2\pi} \quad (H)$$

(assuming ave flux density at average radius r)

Parallel conductors of radius a:

radius a

$$\frac{L}{\ell} = \frac{\mu_0}{\pi} \cosh^{-1}\left(\frac{d}{2a}\right) (H/m)$$

for $d \gg a$: $\frac{L}{\ell} \approx \frac{\mu_0}{\pi} \ln \frac{d}{a}$

cylindrical conductor parallel to ground plane:

$$\frac{L}{\ell} = \frac{\mu_0}{2\pi} \cosh^{-1}\left(\frac{d}{2a}\right) (H/m) \approx \frac{\mu_0}{2\pi} \ln\left(\frac{d}{a}\right) (H/m)$$

long solenoid of small cross-sectional area S:

$$L = \frac{\mu_0 N^2 S}{\ell} \quad (H)$$

Time Harmonic Maxwell's Eqs

$\nabla \times \overline{E} = -j\omega \overline{B}$ $\quad \overline{B} = \mu \overline{H}$

$\nabla \times \overline{H} = j\omega \overline{D} + \overline{J}$, $\quad \overline{D} = \varepsilon \overline{E}$ and [illegible]

$\overline{J} =$ [illegible]

[illegible]

Maxwells Equations

Point Form:	Integral Form:	Name:
$\nabla \times \overline{H} = \overline{J}_c + \frac{\partial \overline{D}}{\partial t}$	$\oint \overline{H} \cdot d\overline{l} = \int_S \left(\overline{J}_C + \frac{\partial \overline{D}}{\partial t} \right) \cdot d\overline{S}$	Ampere's Law
$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$ [illegible]	$\oint \overline{E} \cdot d\overline{l} = \int_S \left(-\frac{\partial \overline{B}}{\partial t} \right) \cdot d\overline{S}$	Faraday's Law
$\nabla \cdot \overline{D} = \rho$	$\oint_S \overline{D} \cdot d\overline{S} = \int_V \rho \, dV$	Gauss' Law
$\nabla \cdot \overline{B} = 0$ [illegible]	$\oint_S \overline{B} \cdot d\overline{S} = 0$ [illegible]	Ma[illegible]